The Long and the Short of the Risk-Return Trade-Off *

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Abstract

The relationship between conditional volatility and expected stock market returns, the so-called risk-return trade-off, has been studied at high- and low-frequency. We propose an asset pricing model with generalized disappointment aversion preferences and short- and long-run volatility risks that captures several stylized facts associated with the risk-return trade-off at short and long horizons. Writing the model in Bonomo et al. (2011) at the daily frequency, we aim at reproducing the moments of the variance premium and realized volatility, the long-run predictability of cumulative returns by the past cumulative variance, the short-run predictability of returns by the variance premium, as well as the daily autocorrelation patterns at many lags of the VIX and of the variance premium, and the daily cross-correlations of these two measures with leads and lags of daily returns. By keeping the same calibration as in this previous paper, we ensure that the model is capturing the first and second moments of the equity premium and the risk-free rate, and the predictability of returns by the dividend ratio. Overall adding generalized disappointment aversion to the Kreps-Porteus specification improves the fit for both the short-run and the long-run risk-return trade-offs.

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1 Introduction

To study the relationship between conditional volatility and expected stock market returns researchers have mainly run linear regressions. The outcome after more than two decades of empirical studies is rather disappointing. Some find a positive relation, others a negative one. In many studies, there is no significant trade-off[1]. Several recent contributions have revived the debate. Bandi and Perron (2008) find that the dependence is statistically mild at short horizons, which explains the contradicting results in the literature, but increases with the horizon and is strong in the long run (between 6 and 10 years). A recent trend in the literature has also put forward the variance risk premium (VRP) as a strong predictor of stock returns in the short run (see in particular Bollerslev et al. (2009)). Bollerslev et al. (2006) study the relationship between volatility and past and future returns in high-frequency equity market data. They find an asymmetric pattern in the cross-correlations between absolute high-frequency returns and current and past high-frequency returns. Correlations between absolute returns and past returns are significantly negative for several days, while the reverse cross-correlations between absolute returns and future returns are negligible.

We propose an equilibrium consumption-based asset pricing model with generalized disappointment aversion preferences and volatility risk to rationalize these recently put-forward stylized facts. We extend the model in Bonomo et al. (2011) in two ways. First, since we want to address short-run volatility-return relationship, we write the model at the daily frequency. Second, we add a short-run volatility risk to the long-run volatility risk in Bonomo et al. (2011). Our main contribution is to reproduce relations between returns and volatility at short and long horizons with an equilibrium model calibrated at a high-frequency daily level. We can then solve the model daily and construct realized quantities at lower frequencies. Thanks to Markov-switching fundamentals, a key advantage of the model proposed by Bonomo et al. (2011) is to find analytical formulas for moments of asset pricing quantities such as payoff ratios and returns, and for coefficients of predictability regressions at any horizon. In this framework we are able to produce analytical results at high frequency as in Bollerslev et al. (2006) together with the long-run regressions of Bandi and Perron (2008) in the same model. For generating empirical stylized facts, we keep the same calibration

[1]In a survey about measuring and modeling variation in the risk-return trade-off, Lettau and Ludvigson (2010) attribute in large part the disagreement in the empirical literature on this relation to the limited amount of information generally used to model the conditional mean and conditional volatility of excess stock market returns. Rossi and Timmermann (2010) argue that there is no theoretical reason for assuming a linear relationship between the expected returns and the conditional volatility. They found support for nonlinear patterns in the risk-return trade-off.

as in Bonomo et al. (2011) to make sure that the model produces first and second moments of price-dividend ratios and asset returns as well as return predictability patterns in line with the data.

At the high-frequency level, we assess the capacity of the model to reproduce the autocorrelation of daily returns, squared daily returns, and cross-correlations of daily returns and squared returns at various leads and lags. We also build measures of monthly realized variance (RV) by summing daily squared returns and compute moments of realized volatility. The variance premium is obtained by first taking the expectation under the risk-neutral measure of this realized volatility and subtracting the latter from the obtained risk-neutral volatility. The predictability of returns by the variance premium is then established for horizons of one to twelve months. At longer horizons, we reproduce the predictability regressions of Bandi and Perron (2008) by aggregating returns and volatilities over periods of one to ten years.

With generalized disappointment aversion preferences, the stochastic discount factor (SDF) has a kink at a disappointing threshold equal to a given fraction of the certainty equivalent of lifetime future utility. In regular disappointment aversion preferences, the threshold is equal to the certainty equivalent. In the model we propose, expected consumption growth is constant. Therefore, the only long-run risk is an economic uncertainty risk captured by the volatility of consumption. To capture a richer short-run dynamics in volatility and the predictability of returns by the variance premium we add a short-lived volatility component to the persistent component that was used in Bonomo et al. (2011). Shocks in the high- and low-persistence components of consumption growth volatility affect the volatility of the SDF. A persistent increase in consumption growth volatility increases the volatility of future utility. A more volatile future utility increases the probability of disappointing outcomes, making the SDF more volatile. In our model dividends share the consumption volatility process, so an increase in volatility will increase negative covariance between the SDF and the equity return, increasing both the equity premium and the stock return volatility. This will ensure that the long-horizon regressions of aggregated returns over aggregated volatilities will produce R^2 that increase with the horizon.

The variance risk premium is measured as the difference between the squared VIX index (VIX^2) and expected realized variance. In the short-run, the low-persistence component of consumption growth volatility will add volatility to the SDF. This additional volatility will impact relatively more the option-implied variance and the volatility of the variance premium will increase. So, we expect a stronger relationship between the variance premium and the future returns in the short-run than in the long-run. Indeed, for the short-run predictability of returns by the variance premium, we obtain with the model the pattern exhibited by the data (a peak around 2 to 3 months and a slow decline up to 12 months).

In terms of moments of the VIX^2, RV and VRP, we match the mean of the VIX^2 but tend to overestimate the mean of the realized volatility, therefore underestimating the mean of the variance premium. For the second moments, we overestimate the standard deviation of both VIX^2 and RV and underestimate the standard deviation of the variance premium. For the short-run risk-return trade-off stylized facts, we are able to reproduce the daily autocorrelation patterns in VIX^2 and VRP, up to 90 lags, that is the more persistent autocorrelation for the first measure and the faster decay for the variance premium. For the cross-correlations of VIX^2 and VRP with 22 leads and lags of daily returns, we observe a negative pattern in the lags and a close to zero pattern in the leads for both measures. Our model produces negative cross-correlations in the lags (interpreted in the literature as a leverage effect), albeit weaker than in the data, but overestimates the positive cross-correlations in the leads. Therefore, our model creates a stronger volatility feedback effect than observed. This short-run predictability of returns by the variance measures remains in the long-run since the model reproduces the increasing explanatory power at longer horizons found by Bandi and Perron (2008).

Given that all moments and regression coefficients are obtained analytically we are able to conduct a thorough comparison between our GDA specification and two important sub-cases. The first one (called DA0) will be the simplest specification among disappointment averse preferences. The threshold is set at the certainty equivalent and we do not allow any curvature in the stochastic discount factor except for the disappointment aversion kink. Therefore, without disappointment, the SDF will be constant and equal to the constant time discount parameter. The second one (denoted by KP) is of course the Kreps-Porteus preferences which are used most often in long-run risk models as in the original Bansal and Yaron (2004). For the three sets of preferences, we compute all asset pricing moments, predictability regression statistics, and high-frequency dynamics autocorrelations and cross-correlations. In addition, we report graphs that exhibit the sensitivity of all statistics to variations in the key persistence values of the two components of consumption growth volatility. This analysis of the interplay between the persistence of the two components produces very interesting patterns in some of the statistics, that in all likelihood will be hard to detect in an estimation exercise. The analytical solutions are very useful to measure the robustness of model implications. In several dimensions, both DA0 and KP come short of matching our benchmark specification in reproducing the moment and predictability statistics. For reproducing asset returns moments and predictability of returns by the dividend-price ratio, pure disappointment aversion as captured by DA0 plays the most important role, as shown in Bonomo et al. (2011). However, for the risk-return trade-off statistics, short-run risk aversion appears to be important. Therefore, GDA preferences, which incorporate both disappointment aversion and short-run risk aversion are better able to reproduce the complete set of stylized facts. For KP preferences, we already pointed out in Bonomo et al. (2011) its inability to capture the predictability of excess returns by the price-dividend ratio, as well as its counterfactual predictability of consumption growth. For risk-return trade-off statistics, KP preferences match poorly the realized variance, VIX^2 and variance premium moments, but reproduce somehow the patterns for the short-run predictability of excess returns by the variance premium, the long-run risk-return trade-off and the daily autocorrelations and cross-correlations. However the magnitudes of the statistics are not in line with the data.

Other recent papers have addressed some of these stylized facts with equilibrium models. Bollerslev et al. (2009) and Drechsler and Yaron (2011) provide a rationalization of the return predictability by the variance premium based on extensions of the Bansal and Yaron (2004) long-run risk model. Both models add in different ways a time-varying volatility of volatility to the initial model where it was constant. A serious limitation of both models is that the model-implied measure of the variance premium is not based on an accumulation of daily quantities as in the data but on monthly conditional measures. Moreover, they do not consider the long-run risk-return trade-off.

Drechsler (2013) builds an equilibrium model with ambiguity aversion to capture properties of index option prices, equity returns, variance, and the risk-free rate[2]. Investors who are afraid about model uncertainty are ready to pay a large premium for index options because they hedge again potential model misspecifications, most notably the presence of jump shocks to cash flow growth and volatility. Time variation in uncertainty generates variance premium fluctuations, helping to explain their power to predict stock returns. In our model, only shocks to consumption

[2]Schreindorfer (2014) aims at explaining the same stylized facts with the Bonomo et al. (2011) GDA model and a heteroscedastic randow walk for consumption with the multifractal process of Calvet and Fisher (2007).

volatility matter and time variation of the variance premium comes from the stochastic nature of the disappointment magnitude as explained before.

Another structural approach is proposed by Bollerslev and Zhou (2005). They provide a theoretical framework for assessing the empirical links between returns and realized volatilities. They show that the sign of the correlation between contemporaneous return and realized volatility depends importantly on the underlying structural parameters that enter nonlinearly in the coefficient.

Several papers have used Markov chains to model consumption growth volatility. Earlier papers include Cecchetti et al. (1990), Bonomo and Garcia (1994) and Bonomo and Garcia (1996). More recently, Calvet and Fisher (2007) have modeled consumption volatility at high and low frequencies with a multi-fractal Markov-witching process.

The rest of the paper is organized as follows. Section 2 sets up the model for both preferences and dynamics of fundamentals and provides the asset pricing solution. In Section 3, we detail the various measures used as stylized facts for the risk-return trade-off and provide the model-based analytical formulas for assessing the trade-off. Section 4 reviews the empirical stylized facts for these various measures over the period 1990-2012 for facts involving the variance risk premium and 1930-2012 for the long-run risk-returns trade-off. We also compute the short-run measures over the period 1990-2007 to account for the potential effect of the financial crisis on these quantities. The calibration and the assessment of the model along the various measures of the risk-return trade-off are reported in Section 5. Section 6 concludes. An online appendix provides the details of the analytical derivations for the asset pricing moments and the risk-return trade-off measures.

2 Model Setup, Assumptions and Asset Pricing Solution

We assume that there are $1/\Delta$ trading periods in a month, and that month t contains the periods $t-1+j\Delta, j=1,2,\ldots,1/\Delta$. For example, $\Delta = 1/22$ for daily periods and $\Delta = 1/(78 \times 22)$ for 5-min interval periods. We refer to the month as the frequency 1 and to the period as the frequency Δ. So defined, the frequency h refers to h months or equivalently h/Δ periods. For example, the frequency 12 corresponds to yearly. We assume that the decision interval of economic agents corresponds to the frequency Δ so that dynamics of preferences, endowments and other exogenous state variables are given at the frequency Δ.

2.1 Equilibrium Consumption and Dividends Growths Dynamics

We assume that equilibrium consumption and dividends growths are unpredictable, at least at the frequency Δ, and that their conditional variance as well as their conditional correlation change according to a Markov variable s_t which takes N values, $s_t \in \{1, 2, \ldots, N\}$, when N states of nature are assumed for the economy. The process s_t evolves according to a transition probability matrix P defined as:

$$P^\top = [p_{ij}]_{1 \le i,j \le N} \quad \text{and} \quad p_{ij} = Prob\,(s_{t+\Delta} = j \mid s_t = i)\,. \tag{1}$$

Let $\zeta_t = e_{s_t}$, where e_j is the $N \times 1$ vector with all components equal to zero but the jth component is equal to one. Therefore, the dynamics of consumption and dividends are given by:

$$\begin{aligned} g_{c,t+\Delta} &= \ln\left(\frac{C_{t+\Delta}}{C_t}\right) = \mu_x + \sigma_t \varepsilon_{c,t+\Delta} \\ g_{d,t+\Delta} &= \ln\left(\frac{D_{t+\Delta}}{D_t}\right) = \mu_x + \nu_d \sigma_t \varepsilon_{d,t+\Delta} \end{aligned} \tag{2}$$

where $\sigma_t^2 = \omega_c^\top \zeta_t$, and where

$$\begin{pmatrix} \varepsilon_{c,t+\Delta} \\ \varepsilon_{d,t+\Delta} \end{pmatrix} \mid \left\{ \varepsilon_{c,j\Delta}, \varepsilon_{d,j\Delta}, j \le \frac{t}{\Delta}; \zeta_{k\Delta}, k \in \mathbb{Z} \right\} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \right), \tag{3}$$

with

$$\rho_t = \frac{1 - \exp\left(-\beta_{\rho 0} - \beta_{\rho\sigma} \ln \sigma_t^2\right)}{1 + \exp\left(-\beta_{\rho 0} - \beta_{\rho\sigma} \ln \sigma_t^2\right)} = \rho^\top \zeta_t. \tag{4}$$

The scalar μ_x is the expected growth of aggregate consumption, which is assumed equal to that of aggregate dividends. The two vectors ω_c and ρ contain state values of the volatility of consumption growth and of the correlation between consumption growth and dividend growth, respectively. The ith element of a vector refers to the value in state $s_t = i$. Equation (4) shows that the conditional correlation between consumption and dividends growths depends on the state of the economy as determined by the volatility of aggregate consumption. In particular if $\beta_{\rho\sigma} = 0$, then consumption and dividends growth correlation is constant; if $\beta_{\rho\sigma} > 0$ this correlation increases with macroeconomic uncertainty.

We assume that the log conditional variance of aggregate consumption growth is given by

$$\ln \sigma_t^2 = a_z + b_{1z} z_{1,t} + b_{2z} z_{2,t} \tag{5}$$

where $z_{1,t}$ and $z_{2,t}$ are two independent two-state Markov chains that can take values 0 and 1, corresponding to a low (L) and a high (H) states. The chain $z_{i,t}$ has the persistence ϕ_{iz}, a nonnegative skewness and the kurtosis k_{iz}. Following Bonomo et al. (2011) its transition matrix P_{iz} may be written

$$P_{iz}^{\top} = \begin{pmatrix} p_{iz,LL} & 1 - p_{iz,LL} \\ 1 - p_{iz,HH} & p_{iz,HH} \end{pmatrix} \text{ with conditional state probabilities given by} \tag{6}$$
$$p_{iz,LL} = \frac{1+\phi_{iz}}{2} + \frac{1-\phi_{iz}}{2}\sqrt{\frac{k_{iz}-1}{k_{iz}+3}} \text{ and } p_{iz,HH} = \frac{1+\phi_{iz}}{2} - \frac{1-\phi_{iz}}{2}\sqrt{\frac{k_{iz}-1}{k_{iz}+3}}.$$

The kurtosis of the two-state Markov chain $z_{i,t}$ fully characterizes its stationary distribution, for which the unconditional state probabilities are given by

$$\begin{aligned} \pi_{iz,L} &= P\left(z_{i,t} = 0\right) = \frac{1 - p_{iz,HH}}{2 - p_{iz,LL} - p_{iz,HH}} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{k_{iz}-1}{k_{iz}+3}} \\ \pi_{iz,H} &= P\left(z_{i,t} = 1\right) = \frac{1 - p_{iz,LL}}{2 - p_{iz,LL} - p_{iz,HH}} = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{k_{iz}-1}{k_{iz}+3}}, \end{aligned} \tag{7}$$

and the skewness is given by

$$s_{iz} = \frac{\pi_{iz,L} - \pi_{iz,H}}{\sqrt{\pi_{iz,L}\pi_{iz,H}}} = \sqrt{k_{iz} - 1}. \tag{8}$$

The combination of the two states of $z_{1,t}$ and the two states of $z_{2,t}$ leads to four distinct states for the economy: $LL \equiv 1$, $LH \equiv 2$, $HL \equiv 3$ and $HH \equiv 4$. By the independence of the chains $z_{1,t}$ and $z_{2,t}$, the transition probability matrix associated with the four states of the economy also derives easily as $P = P_{1z} \otimes P_{2z}$. The state values of the conditional variance of aggregate consumption growth are given by $\omega_c = \begin{pmatrix} \exp\left(a_z\right) & \exp\left(a_z + b_{2z}\right) & \exp\left(a_z + b_{1z}\right) & \exp\left(a_z + b_{1z} + b_{2z}\right) \end{pmatrix}^{\top}$.

The logarithm of conditional variance $\ln \sigma_t^2$ has the mean μ_σ, the volatility σ_σ and the skewness s_σ that we want to match with the coefficients a_z, $b_{1,z}$ and $b_{2,z}$ in equation (5). We also assume that the first component $z_{1,t}$ has a zero skewness ($s_{1z} = 0$) which for a two-state Markov chain is also equivalent to a unitary kurtosis ($k_{1z} = 1$) and constant conditional volatility (homoscedasticity).

Given ϕ_{1z}, ϕ_{2z} and κ_{2z}, we solve for a_z, $b_{1,z}$ and $b_{2,z}$ to match the mean μ_σ, the volatility σ_σ and the skewness s_σ of $\ln \sigma_t^2$. We find that

$$
\begin{aligned}
& b_{1z} = \frac{\sigma_\sigma}{\sqrt{\pi_{1z,1}\pi_{1z,2}}} \left(1 - \left(\frac{s_\sigma}{s_{2z}} \right)^{2/3} \right)^{1/2} \quad \text{and} \quad b_{2z} = \frac{\sigma_\sigma}{\sqrt{\pi_{2z,1}\pi_{2z,2}}} \left(\frac{s_\sigma}{s_{2z}} \right)^{1/3} \\
& a_z = \mu_\sigma - b_{1z}\pi_{1z,2} - b_{2z}\pi_{2z,2}.
\end{aligned}
\tag{9}
$$

Equation (9) implies that $s_\sigma < s_{2z}$, or equivalently $\kappa_{2z} > 1 + s_\sigma^2$. Later in our calibration analysis, we assume that the first component is very persistent, with $\phi_{1z}^{1/\Delta}$ close to one, and that the second component is not persistent, with $\phi_{2z}^{1/\Delta}$ typically less than 0.9.

Several papers including Alizadeh et al. (2002), Barndorff-Nielsen and Shephard (2001), Chernov et al. (2003), and Meddahi (2001) highlighted the importance of having two factors driving the volatility process for models of daily returns. Typically, one factor will be very persistent in order to capture persistence in volatility while the second one will be less persistent but very volatile. The second factor is a way to capture fat tails in returns that a one-factor model with Gaussian errors will not capture; see Meddahi (2001) for more details. In our model, the two independent two-state Markov chains will play the role of these two factors, one being very persistent at the daily level and one being less persistent but more volatile with a large kurtosis.

2.2 Preferences

The representative investor has generalized disappointment aversion (GDA) preferences of Routledge and Zin (2010). Following Epstein and Zin (1989), such an investor derives utility from consumption, recursively as follows:

$$
\begin{aligned}
V_t &= \left\{ (1-\delta) \, C_t^{1-\frac{1}{\psi}} + \delta \left[\mathcal{R}_t \left(V_{t+\Delta} \right) \right]^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad \text{if } \psi \neq 1 \\
&= C_t^{1-\delta} \left[\mathcal{R}_t \left(V_{t+\Delta} \right) \right]^{\delta} \quad \text{if } \psi = 1.
\end{aligned}
\tag{10}
$$

The current period lifetime utility V_t is a combination of current consumption C_t, and $\mathcal{R}_t \left(V_{t+\Delta} \right)$, a certainty equivalent of next period lifetime utility. With GDA preferences the risk-adjustment function $\mathcal{R}(\cdot)$ is implicity defined by:

$$\frac{\mathcal{R}^{1-\gamma}-1}{1-\gamma}=\int_{-\infty}^{\infty}\frac{V^{1-\gamma}-1}{1-\gamma}dF(V)-\ell\int_{-\infty}^{\theta\mathcal{R}}\left(\frac{(\theta\mathcal{R})^{1-\gamma}-1}{1-\gamma}-\frac{V^{1-\gamma}-1}{1-\gamma}\right)dF(V), \tag{11}$$

where $\ell \geq 0$ and $0 < \theta \leq 1$. When ℓ is equal to zero, $\mathcal{R}$ becomes the Kreps and Porteus (1978) preferences, while V_t represents Epstein and Zin (1989) recursive utility. When $\ell > 0$, outcomes lower than $\theta\mathcal{R}$ receive an extra weight ℓ, decreasing the certainty equivalent. Thus, the parameter ℓ is interpreted as a measure of disappointment aversion, while the parameter θ is the percentage of the certainty equivalent $\mathcal{R}$ such that outcomes below it are considered disappointing[3]. Equation (11) makes clear that the probabilities to compute the certainty equivalent are redistributed when disappointment sets in, and that the threshold determining disappointment is changing over time.

With KP preferences, Hansen et al. (2008) derive the stochastic discount factor in terms of the continuation value of utility of consumption, as follows:

$$M^*_{t,t+\Delta}=\delta\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+\Delta}}{\mathcal{R}_t\left(V_{t+\Delta}\right)}\right)^{\frac{1}{\psi}-\gamma}=\delta\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\frac{1}{\psi}}Z_{t+\Delta}^{\frac{1}{\psi}-\gamma}, \tag{12}$$

where

$$Z_{t+\Delta}=\frac{V_{t+\Delta}}{\mathcal{R}_t\left(V_{t+\Delta}\right)}=\left(\delta\left(\frac{C_{t+\Delta}}{C_t}\right)^{-\frac{1}{\psi}}R_{c,t+\Delta}\right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{13}$$

and where the second equality in Eq. (13) implies an equivalent representation of the stochastic discount factor given in Eq. (12), based on consumption growth and the gross return $R_{c,t+\Delta}$ to a claim on future aggregate consumption stream. In general this return is unobservable. The return to a stock market index is sometimes used to proxy for this return as in Epstein and Zin (1991); or other components can be included such as human capital with assigned market or shadow values. If $\gamma = 1/\psi$, Eq. (12) corresponds to the stochastic discount factor of an investor with time-separable utility and constant relative risk aversion, where the powered consumption growth values short-run consumption risk as usually understood. The ratio of future utility $V_{t+\Delta}$ to the certainty equivalent of this future utility $\mathcal{R}_t\left(V_{t+\Delta}\right)$ will add a premium for long-run consumption risk as put forward by Bansal and Yaron (2004) and measured by Hansen et al. (2008).

For GDA preferences, long-run consumption risk enters in an additional term capturing disap-

[3]Notice that the certainty equivalent, besides being decreasing in γ, is also decreasing in ℓ (for $\ell \geq 0$), and decreasing in θ (for $0 < \theta \leq 1$). Thus ℓ and θ are also measures of risk aversion, but of different types than γ.

pointment aversion[4], as follows:

$$M_{t,t+\Delta} = M^{*}_{t,t+\Delta} \left(\frac{1 + \ell I \left(Z_{t+\Delta} < \theta\right)}{1 + \ell \theta^{1-\gamma} E_t \left[I \left(Z_{t+\Delta} < \theta\right)\right]} \right), \tag{14}$$

where $I(\cdot)$ is an indicator function that takes the value 1 if the condition is met and 0 otherwise.

2.3 Asset Pricing Solution

In this model, we can solve for asset prices analytically, for example the price-dividend ratio $P_{d,t}/D_t$ (where $P_{d,t}$ is the price of the portfolio that pays off equity dividend), the price-consumption ratio $P_{c,t}/C_t$ (where $P_{c,t}$ is the price of the unobservable portfolio that pays off consumption) and the price $P_{f,t}/1$ of the one-period risk-free bond that delivers one unit of consumption. To obtain these asset prices, we need expressions for $\mathcal{R}_t\left(V_{t+\Delta}\right)/C_t$, the ratio of the certainty equivalent of future lifetime utility to current consumption, and for V_t/C_t, the ratio of lifetime utility to current consumption. The Markov property of the model is crucial for deriving analytical formulas for these expressions and we adopt the following notation:

$$\frac{\mathcal{R}_t\left(V_{t+\Delta}\right)}{C_t} = \lambda_{1z}^{\top}\zeta_t, \quad \frac{V_t}{C_t} = \lambda_{1v}^{\top}\zeta_t, \quad \frac{P_{d,t}}{D_t} = \lambda_{1d}^{\top}\zeta_t \text{ and } P_{f,t} = \lambda_{1f}^{\top}\zeta_t. \tag{15}$$

Solving these ratios amounts to characterize the vectors λ_{1z}, λ_{1v}, λ_{1d} and λ_{1f} as functions of the parameters of the consumption and dividends, dynamics and of the recursive utility function defined above. In Section A of the online appendix, we provide explicit analytical expressions for these ratios.

We use results from Bonomo et al. (2011) to show that the excess log equity return over the risk-free rate $r_{t+\Delta}$ can also be written as

$$r_{t+\Delta} = \zeta_t^{\top}\Lambda\zeta_{t+\Delta} + \sqrt{\omega_d^{\top}\zeta_t}\,\varepsilon_{d,t+\Delta}, \tag{16}$$

[4] Although Routledge and Zin (2010) do not model long-run consumption risk as it is done in Bansal and Yaron (2004), they discuss how its presence could interact with GDA preferences in determining the marginal rate of substitution.

where the components of matrix Λ are explicitly defined by

$$\nu_{ij} = \ln\left(\frac{\lambda_{1d,j}+1}{\lambda_{1d,i}}\right) + \mu_{d,i} + \ln\lambda_{1f,i}. \tag{17}$$

3 The Risk-Return Trade-offs

The model just described implies a complex nonlinear relationship between the consumption volatility risks and future returns on the dividend-paying asset. However, as often in assessing the soundness of a consumption-based asset pricing model, it is useful and instructive to measure its capacity to reproduce some simple stylized facts such as moments and regression statistics. For the risk-returns trade-offs, we retain the long-run regressions proposed by Bandi and Perron (2008) whereby cumulated returns over long horizons are regressed on lagged, cumulated returns volatilities over the same horizon. Since we write our model at the daily frequency, we are able to construct the equivalent of the empirical measures of these quantities. The daily frequency allows us also to build short-run statistics that have been used to characterize the short-run dynamics of returns volatility and the short-run risk-return trade-off. We look in particular at daily cross-correlations between returns and volatility at 22-day leads and lags, as well as at the monthly predictability of future returns by the variance premium, up to twelve months. For all these statistics we provide the analytical formulas implied by our model.

3.1 The Long-Run Risk-Return Trade-Off

Following Bandi and Perron (2008) who examine the predictability of future long-horizon excess returns by past long-horizon realized variance, we define one-period excess log returns and realized variance by

$$r_{t,t+1} = \sum_{j=1}^{1/\Delta} r_{t+j\Delta} \quad \text{and} \quad \sigma^2_{t-1,t} = \sum_{j=1}^{1/\Delta} r^2_{t-1+j\Delta}, \tag{18}$$

and also aggregate values over multiple periods as

$$r_{t,t+h} = \sum_{l=1}^{h} r_{t+l-1,t+l} \quad \text{and} \quad \sigma^2_{t-m,t} = \sum_{l=1}^{m} \sigma^2_{t-l,t-l+1}. \tag{19}$$

Notice that in the empirical investigation of Bandi and Perron (2008), monthly excess returns and realized variance are based on daily returns, thus corresponding to $\Delta = 1/22$. Furthermore, their realized variance is based on nominal returns and not on real returns, due to the unavailability of daily inflation data. Notice that nominal excess log returns over the log risk-free return are identical to their real counterparts since inflation rate cancels out in the subtraction. To the contrary, we measure realized variance using excess returns as we do not explicitly model inflation. In principle, this would lead to minor differences in empirical studies.

We consider the following regression:

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{mh} \frac{\sigma^2_{t-m,t}}{m} + \epsilon^{(m)}_{t,t+h}, \tag{20}$$

for which population values of the intercept α_{mh}, the slope coefficient β_{mh} and the coefficient of determination R^2_{mh} are given by

$$\begin{aligned} \alpha_{mh} &= \frac{E\left[r_{t,t+h}\right]}{h} - \beta_{mh} \frac{E\left[\sigma^2_{t-m,t}\right]}{m} \\ \beta_{mh} &= \frac{m}{h} \frac{Cov\left(\sigma^2_{t-m,t}, r_{t,t+h}\right)}{Var\left[\sigma^2_{t-m,t}\right]} \quad \text{and} \quad R^2_{mh} = \frac{Cov\left(\sigma^2_{t-m,t}, r_{t,t+h}\right)^2}{Var\left[\sigma^2_{t-m,t}\right] Var\left[r_{t,t+h}\right]}. \end{aligned} \tag{21}$$

In the context of the equilibrium asset pricing model described in Section 2, we provide analytical formulas for the population values defined in Eq. (21). These quantities are relevant for assessing the risk-return relation through the predictability regression (20). Expressions for the expected values, variances and covariances in equation (21) are provided in sections C and D of the online appendix.

3.2 Realized Variance, Variance Premium and Short-run Predictability of Returns

In this paper, we will consider two different definitions of the variance risk premium. The first one is given by

$$vp^{(1)}_t \equiv E^{\mathbb{Q}}_t \left[\sigma^2_{r,t+1}\right] - E_t\left[\sigma^2_{r,t+1}\right] \quad \text{where} \quad \sigma^2_{r,t} \equiv Var_t\left[r_{t,t+1}\right]. \tag{22}$$

This definition, considered in the theoretical framework of Bollerslev et al. (2009), does not have a model-free counterpart. Consequently, these authors made a couple of changes in their empirical application. In order to measure the first term in Eq. (22), they used the (square of the) VIX measure which is the conditional expectation of the integrated variance under the $\mathbb{Q}$-measure when one assumes a continuous time framework (see for instance Bollerslev et al. (2012)). When there is no drift, the conditional expectation of the integrated variance equals the conditional expectation of the realized variance computed as the sum of the squared returns whatever the discretization sampling (Meddahi (2002) and Andersen et al. (2005)). This leads Bollerslev et al. (2009) to measure the second part of Eq. (22) as the expected value of the realized variance of a future period. This is why we adopt a second definition of the variance risk premium given by

$$vp_t^{(2)} \equiv E_t^{\mathbb{Q}}\left[\sigma_{t,t+1}^2\right] - E_t\left[\sigma_{t,t+1}^2\right] \quad \text{where} \quad \sigma_{t,t+1}^2 \equiv \sum_{j=1}^{1/\Delta} r_{t+j\Delta}^2. \tag{23}$$

The analytical formula implied by our equilibrium model is given in the following proposition.

Proposition 3.1 *One has*

$$vp_t^{(1)} = \lambda_{vp}^{(1)\top}\zeta_t \quad \text{with} \quad \lambda_{vp}^{(1)} = \Upsilon_{1/\Delta}^{\mathbb{Q}} - \Upsilon_{1/\Delta}, \tag{24}$$

where the vectors $\Upsilon_{1/\Delta}^{\mathbb{Q}}$ *and* $\Upsilon_{1/\Delta}$ *are given in Eq. (E.11) of the online appendix.*

Likewise, one has

$$vp_t^{(2)} = \lambda_{vp}^{(2)\top}\zeta_t \quad \text{with} \quad \lambda_{vp}^{(2)} = \lambda_{vix} - \lambda_{rv}, \tag{25}$$

and

$$\lambda_{vix} = \left(\sum_{j=1}^{1/\Delta} \Psi_{j-1}^{\mathbb{Q}(2)}\right)^{\top} \quad \text{and} \quad \lambda_{rv} = \left(\sum_{j=1}^{1/\Delta} P^{j-1}\right)^{\top} \Psi_0^{(2)},$$

where the vectors $\Psi_{j-1}^{\mathbb{Q}(2)}$ *for* $j = 1, ..., 1/\Delta$, *and* $\Psi_0^{(2)}$ *are given in Eq. (E.9) and Eq. (D.6) respectively of the online appendix.*

One can now easily characterize the ability of the two measures of variance risk premia to predict future returns given that one can write, for $i = 1$ or 2,

$$\frac{r_{t,t+l}}{l} = \alpha_{kl}^{(i)} + \beta_{1,kl}^{(i)} vp_{t-k}^{(i)} + \epsilon_{t,t+l}^{(i,k)}, \tag{26}$$

for which population values of the intercept $\alpha_{kl}^{(i)}$ the slope $\beta_{kl}^{(i)}$ and the coefficient of determination $R_{i,kl}^2$ are given by

$$\alpha_{kl}^{(i)} = \frac{E\left[r_{t,t+l}\right]}{l} - \beta_{1,kl}^{(i)} E\left[vp_{t-k}^{(i)}\right], \quad \beta_{kl}^{(i)} = \frac{1}{l}\frac{Cov(vp_{t-k}^{(i)}, r_{t,t+l})}{Var[vp_{t-k}^{(i)}]} \text{ and } R_{i,kl}^2 = \frac{\left(Cov(vp_{t-k}^{(i)}, r_{t,t+l})\right)^2}{Var[vp_{t-k}^{(i)}]Var\left[r_{t,t+l}\right]}.$$

One can show that

$$\begin{aligned} E\left[vp_{t-k}^{(i)}\right] &= \lambda_{vp}^{(i)\top}\mu^{\zeta} \quad \text{and} \quad Var\left[vp_{t-k}^{(i)}\right] = \lambda_{vp}^{(i)\top}\Sigma^{\zeta}\lambda_{vp}^{(i)} \\ Cov\left(vp_{t-k}^{(i)}, r_{t,t+l}\right) &= \sum_{j=1}^{l/\Delta}\left(\Psi_0^{(1)}\right)^{\top} P^{k/\Delta+j-1}\Sigma^{\zeta}\lambda_{vp}^{(i)}. \end{aligned} \tag{27}$$

The formulas for measuring the daily autocorrelations of the implied variance and variance premium measures and their cross-correlations with returns (leverage and volatility feedback effects) are provided in Section D of the online appendix.

4 Empirical Stylized Facts

The main goal of this paper is to reproduce risk-return trade-off statistics at both short and long horizons. Daily stylized facts are captured in Figure 1. Panel A1 depicts the daily autocorrelations of the three variance measures up to a lag length of 90 days . The VIX^2 represents the option-embedded expectation of the cumulative variation of the S&P 500 index over the next month plus a potential variance premium for bearing the corresponding volatility risk. The RV line captures the daily autocorrelations of the realized variance over the next month. The realized variance is computed either as the sum of the daily squared returns over the next 22 days ($SQFor$) or the sum of the daily realized variances over the next 22 days ($RVFor$). Finally, the variance premium (VP) is obtained by projecting the realized variance on variables known at time t and subtracting the predicted value from the VIX^2. This is to capture the difference between the risk-neutral and the objective expectations of the forward integrated variance. The daily VIX^2 is the most persistent, while the variance premium shows a faster decay especially the one obtained with $SQFor$ measure of RV. Panel A2 exhibits the cross-correlations of the VIX^2 and the VP series with daily returns at up to 20 leads and lags. In the left part of the graphs in Panel A2 we observe mainly negative correlations between lagged returns and current measures of variance. This effect dubbed the leverage effect following Black and Cox (1976) is well documented in the empirical literature on volatility (see detailed references in Bollerslev et al. (2012)). The mostly positive cross-correlations in the right part of the figure, which indicate the relation between current volatility and future returns, capture what has been referred to as the volatility feedback effect.

The mean and standard deviation of the three variance series (VIX^2, RV and VP) are reported in Table 1 at both the daily and monthly frequencies. For the short sample excluding the financial crisis, 1990 to 2007, the mean and standard deviation of the variance premium are respectively 11 and 15 percent for the $SQfor$ measure[5]. This is the result of a mean of 33 percent and a standard deviation of close to 24 percent for VIX^2 and a mean of 22 percent and a standard deviation of 23 percent for the realized variance (RV). To compute the expectation of RV under the objective measure at the daily frequency we use the HAR model (see Corsi (2009)) to project the realized variance on past information, as in Bollerslev et al. (2012). Not surprisingly then our values for the moments are quite close to theirs[6]. For the monthly moments, the expectation of the realized variance is based on either on the lagged past value or on the projection on one, two or three lags of the realized variance. The monthly values for the moments of the variance premium are a bit higher than their daily counterparts for both samples.

Another set of stylized facts relates the variance premium to future returns at a lower frequency than daily but still considered short-run. Table 2 reports monthly regression results from one to twelve months. The magnitudes of the predictability varies from the daily to the monthly frequency, form the pre-crisis to the post-crisis sample, and to the method used to compute the variance premium. However, in most cases we observe the same pattern. The R^2 peaks at three months and then declines monotonically up to 12 months to become often negligible. Predictability is stronger at the monthly frequency than at the daily frequency. These patterns have been reported by Bollerslev et al. (2009).

Finally, we consider in Table 3 the long-run risk-return trade-off put forward by Bandi and Perron (2008). They show that the dependence between excess market returns and past market variance increases with the horizon and is strong in the long run, that is between 6 and 10 years. For their sample, from 1952 to 2006, they find R^2 of 26 percent for returns and variances computed

[5]The $RVfor$ measure mainly increases the mean of the realized variance and therefore of the variance premium.
[6]The difference comes from the fact that the series for the realized variance is not exactly the same

over 6 years and up to 73 percent for 10 years. In the long sample we selected, from 1930 to 2012, we observe the same increasing pattern in R^2 but their values are much more modest. For 8, 9 and 10 years, the values are 4, 6 and 15 percent.

In Bonomo et al. (2011), we introduced a similar model at the monthly frequency to match a number of asset pricing moments and predictability statistics. Even though we have enriched the volatility process we still want to reproduce these stylized facts, so that we keep the long-run risk features of the model. The values for the period 1930-2012 are reported in Table 4. The values for the moments of consumption growth are the usual ones with a mean and volatility of around 2 percent. The volatility of dividend growth is of course higher at around 13 percent while the mean is around 1 percent. We observe a correlation of 0.50 between the two growth rate series. The mean of the log equity premium is close to 5 percent, while the risk-free rate mean is close to 1 percent and its volatility around 4 percent. The volatility of excess returns is around 20 percent. In terms of predictability of future returns by the price-dividend ratio, the R^2 is increasing from 3.5 percent at one year to 23 percent at 5 years.

5 Model Calibration and Risk-Return Tradeoff Implications

The challenge is to reproduce the previous stylized facts at high and low frequencies with the same parameters for both preferences and fundamentals. First, we will explain our calibration and then we will assess the capacity of the model described in the previous sections to match the empirical facts.

5.1 Calibration

The model is calibrated at the daily frequency with $\Delta = 1/22$, and daily parameter values are derived from monthly values used in Bansal et al. (2012) and Bonomo et al. (2011). The unconditional mean of monthly consumption growth is $\mu_x^M = 0.15 \times 10^{-2}$. The corresponding daily value is $\mu_x = \mu_x^M \Delta$. The unconditional mean and standard deviation of monthly conditional variance of consumption growth are $\sqrt{\mu_\sigma^M} = 0.7305 \times 10^{-2}$ and $\sigma_\sigma^M = 0.6263 \times 10^{-4}$. The unconditional mean and standard deviation of daily logarithmic conditional variance of consumption growth are then set as $\mu_\sigma = \ln\left(\mu_\sigma^M \Delta\right) - \left(\sigma_M^2 \Delta\right)/2$ and $\sigma_\sigma = \ln\left(1 + \left(\left(\sigma_\sigma^M \sqrt{\Delta}\right) / \left(\mu_\sigma^M \Delta\right)\right)^2\right)$. The monthly persistence of the conditional variance of consumption growth is $\phi^M = 0.995$ in Bonomo et al. (2011); we use this value for the more persistent component of the daily log conditional variance $z_{1,t}$ and assume $\phi_{1z} = 0.995$. Our base case values for the persistence and the kurtosis of the second component $z_{2,t}$ are $\phi_{2z}^{1/\Delta} = 0.50$ and $k_{2z} = 10$, while we assume the base case value of 0.125 for the ratio s_σ / s_{2z}. The parameter $\beta_{\rho\sigma}$ controls the conditional correlation between consumption and dividend growth rates. We set it at 0.07, implying that this correlation increases with macroeconomic uncertainty. For preferences, we keep the same parameters as in Bonomo et al. (2011), where we justify these calibrated values by referring to previous studies where these parameters were estimated.

5.2 Asset Pricing and Risk-Return Tradeoff Model Implications

5.2.1 Asset Pricing Moments and Return Predictability by the Dividend Price ratio

We collect in Table 4 the moments associated with the fundamentals (consumption and dividend growth) as well as with the equity and risk-free rates of return for the benchmark scenario we described in the calibration section above, and for the three sets of preferences GDA, DA0 and KP introduced earlier. We can see that the consumption and dividend processes are closely matched by our calibrated Markov switching process. For asset prices, we consider a set of moments, namely the expected value and the standard deviation of the equity excess returns, the real risk-free rate, and the price-dividend ratio. Overall, the GDA specification fits most moments well except the volatility of the price-dividend ratio (0.27 instead of 0.45 in the data) and the excess equity return (8.62 instead of 5.35 in the data)[7]. The mean and standard deviation of the risk-free rate are particularly well matched. The predictability of excess returns by the price dividend ratio is also well matched in terms of R^2. The simple disappointment aversion specification (DA0) fares also well for these moments and for predictability as put forward in Bonomo et al. (2011). For KP preferences, the main shortcomings are the very low volatility of the price-dividend ratio, which translates into very weak predictability of excess returns by the price-dividend ratio. In Figure 1 included in the online appendix, we report the sensitivity of these results for the GDA specification to variations in two persistence parameters of the volatility process ϕ_{1z} and ϕ_{2z}. First, looking at ϕ_{1z}, we observe that the most affected statistics are the volatility of the price-dividend ratio,

[7]For the mean equity premium, we have to remember that the parameters were calibrated on the post-war data.

and the R^2 of the predictability regressions. A more persistent process will increase these two quantities. For ϕ_{2z}, we observe mainly a level effect on all statistics and a high sensitivity to a lowering of its value. A 0.1 reduction to a value of 0.4 makes the price-dividend and the risk-free rate increase significantly but lowers their volatilities. Therefore, it implies a lowering of the risk premium and of the predictability R^2.

5.2.2 Variance Premium Moments

The variance premium moments are reported in Table 5. In Section 3.3, we have described two methods to compute the model equivalents of the VIX^2 and the realized variance. The first one, referenced as $1st$ in the table, relies on computing the risk neutral and the objective expectations of the conditional variance of returns. The second approach, referenced as $2nd$ in the table, is based on the risk neutral and objective expectations of the monthly sum of daily returns. For GDA, the first method produces moments that match rather well the values for the VIX^2 and the mean of RV but the RV variance is too high. This results in a reasonable value for the model-produced mean of variance premium but too low a value for the standard deviation. The second approach produced a much higher mean for the variance premium, which is more in line with the $RVFor$ empirical way to compute the expectation of the variance premium (20.47 in Table 1), and a very low standard deviation. As pointed out in Section 3.3, the prediction of the monthly realized variance based on squared daily returns (SQFor) equals the prediction the monthly realized variance based on daily realized variances (RVFor) when there is no drift. However, Table 5 highlights substantial differences, which indicates that the drift plays a role when one considers long periods of time like a month. However, one can see in Figure 2 how sensitive the volatility of the VRP (2nd) is to a small change in ϕ_{1z}. The very low value that we found is close to a minimum. Increasing or decreasing a bit the persistence of the first volatility component will increase significantly the volatility of the variance premium.

For the variance premium moments of DA0 in Table 5 the main shortcoming comes from the moments of RV which translate directly to very low mean and standard deviation of the variance premium. For KP preferences, all moments are poorly matched. Going back to Figure 2 one can see it is VRP(2nd) statistics that are most sensitive to a change in ϕ_{1z}. For ϕ_{2z}, the patterns are similar but the levels of the moments vary significantly when ϕ_{2z} is equal to 0.4.

5.2.3 Short-Run and Long-Run Risk-Return Trade-Offs

In this section we will look in turn at the short-run predictability of returns by the variance premium and at the long-run predictability of returns by the past cumulative variance.

Table 6 reproduces the regression results of future returns on the current variance premium. The GDA specification produces a pattern similar to the one in the data for the coefficients of determination, that is a peak in the R^2 at the two- or three-month interval and a monotonic decrease up to 12 months. However, the magnitude of the R^2 is lower than in the data. One important shortcoming of the model, in line with our low estimate of the standard deviation of the variance premium, is the large valued of the slope coefficients. However the values decrease monotonically as in the data. The KP model produces also this pattern but the R^2 are even lower than for the GDA model. Simple disappointment aversion (DA0) does not at all reproduces the empirical pattern.

Figure 3 reports the variations of the regression coefficients and the R^2 for the various horizons as we vary both ϕ_{1z} and ϕ_{2z}. The patterns are quite surprising. For both statistics there is abell pattern with a maximum on either side of our benchmark value of 0.995 for ϕ_{1z}, depending on the value of ϕ_{2z}. The only exception is the monotonically decreasing pattern for the 0.4 value of ϕ_{2z}. Therefore, despite the fact that the empirical patterns are reproduced, the values of the statistics are quite sensitive to the values of the volatility persistence parameters. Therefore, it appears quite challenging to capture with precision these key parameters by usual moment-based estimation procedures.

In Table 7, we report the regression coefficients and the R^2 of the long-run risk-return trade-off regressions, that is the regressions of cumulative returns for a number of months (from 12 to 120) over the cumulative realized volatility for the same number of months. Again the pattern in the data, increasing R^2 as the horizon lengthens, is well captured by the GDA model. Therefore we provide a model for rationalizing the empirical fact put forward by Bandi and Perron (2008). The risk-return trade-off is hard to find in the short-run but comes out clearly in the long run. Both DA0 and KP produce similar patterns but the R^2 values are much smaller.

Figure 4 reports the variations of the regression coefficients and the R^2 for the various horizons as we vary both ϕ_{1z} and ϕ_{2z}. Patterns are monotonically increasing with persistence for both the coefficients and the R^2 up to a high value of ϕ_{1z}, close to 0.995. From that point on, they start to decrease.

5.2.4 High-frequency Dynamics

We have left for the end perhaps the most challenging stylized facts, the autocorrelations of the daily measures of the risk-neutral expectation of the integrated variance and of the variance premium, as well as their daily cross-correlations with returns. This is the risk-return trade-off at the high-frequency level. Even though we write the model at a daily frequency, the model has been initially conceived as a long-run risk model. The only difference with the original model in Bonomo et al. (2011) has been the addition in the volatility process of consumption growth of a less persistent component. Figure 1 plots the model-implied autocorrelations in Panels B1, C1 and D1 on the left hand side and the model-implied cross-correlations on the right hand side in Panel B2, C2 and D2, for GDA, DA0 and KP preferences respectively. The model equivalent of VIX^2 is more persistent than the variance premium, as in the data, but it shows a slightly faster decay than in the data. The variance premium autocorrelation starts near 1 and goes to 0.1 at 90 lags, very much like in the data. KP preferences produce a similar pattern in Panel D1, while for DA0 VIX^2 appears less persistent than VRP in Panel C1. For the cross-correlations, the two variance measures are slightly in the negative to the left, and therefore exhibit a small leverage effect, while they jump back to the positive to the right and show a decreasing pattern while remaining in the positive. This is also consistent with the data for the variance risk premium, but not so much for the VIX^2, where the feedback effect is negligible. The three sets of preferences produce a similar pattern, but the magnitude of the effects differ.

5.2.5 Robustness to the value of the elasticity of intertemporal substitution

The value of the elasticity of intertemporal substitution ψ is a matter of debate. Bansal and Yaron (2004) argue for a value larger than 1 for this parameter since it is critical for reproducing the asset pricing stylized facts. Given this debate over the value of the elasticity of substitution ψ, we set it at 0.75. We maintain for the other parameters the same values as in the benchmark model. In the online appendix we include the graphs corresponding to the sensitivity analysis with respect to the values of phi_{1z} and phi_{2z} for the asset pricing moments, the variance premium moments, the short-run risk-return trade-off and the long-run risk-return trade-off in Figures 2 to 5 respectively. These figures should be compared to the corresponding figures with our benchmark GDA specification with ψ greater than one (equal to 1.5). A careful comparison shows that the value of ψ does not change at all the patterns for the various statistics, while it may affect marginally their magnitudes.

6 Conclusion

We have assessed the ability of a long-run risk equilibrium where preferences display generalized disappointment aversion Routledge and Zin (2010) to capture various stylized facts, high-frequency, short-run and long-run, about the risk-return trade-off in addition to the usual asset pricing moments and the return predictability by the dividend-price ratio. We have therefore written the model developed in Bonomo et al. (2011) at the daily frequency and derived closed-form formulas for all these stylized facts. For the dynamics of the consumption growth process we have maintained a random walk in consumption with a stochastic volatility that includes two mean-reverting components, one much more persistent than the other. Moreover we maintain the same calibration as in Bonomo et al. (2011) for the preference parameters.

Overall, our results are quite supportive of the model. We manage to match rather well most empirical facts, moments as well as predictability patterns, for both asset pricing and risk-return trade-off statistics at all horizons. We observe that pure disappointment aversion is not enough to capture most risk-return trade-off statistics, contrary to what we concluded in Bonomo et al. (2011) for asset pricing moments and return predictability by the price-dividend ratio. Therefore, both disappointment aversion and short-run risk aversion play a role in explaining risk-return trade-off stylized facts.

A remaining challenge concerns the variance of the variance risk premium, which is too low in our model. We could of course find a calibration that does better in that dimension but it will be at the expense of other stylized facts. We will leave this difficult task for future work.

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Table 1: Variance Premium Moments

The entries of the table are the first and second moments of the variance premium, the option-implied variance and the realized variance. In computing the daily variance premium, expected realized variance is a statistical forecast of realized variance using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The realized variance is the sum of squared 5-minute (RVFor) or the sum of squared daily (SQFor) log returns of the S&P 500 index over a 22-day period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$). In computing the monthly variance premium, expected realized variance is simply the lag realized variance or a statistical forecast of realized variance using an $AR(p)$ model. The realized variance is the sum of squared 5-minute log returns of the S&P 500 index over the period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$). All measures are on a monthly basis in percentage-squared.

Full Sample: January 1990 to December 2012									
Daily data				Monthly data					
Moments	RVFor		SQFor	Moments	Lag	$AR(1)$		$AR(2)$	$AR(3)$
$E[VRP]$	20.47		9.73	$E[VRP]$	18.41	18.40		18.40	18.40
$\sigma[VRP]$	22.08		20.19	$\sigma[VRP]$	20.40	19.89		26.87	31.36
$AC1(VRP)$	0.864		0.768	$AC1(VRP)$	0.254	0.555		0.620	0.615
$E[VIX^2]$		39.84		$E[VIX^2]$			39.79		
$\sigma[VIX^2]$		40.23		$\sigma[VIX^2]$			35.72		
$AC1(VIX^2)$		0.971		$AC1(VIX^2)$			0.804		
$E[RV]$	19.37		30.11	$E[RV]$			21.39		
$\sigma[RV]$	33.76		52.53	$\sigma[RV]$			37.60		
$AC1(RV)$	0.997		0.994	$AC1(RV)$			0.649		
Subsample: January 1990 to October 2007									
Daily data				Monthly data					
Moments	RVFor		SQFor	Moments	Lag	$AR(1)$		$AR(2)$	$AR(3)$
$E[VRP]$	18.78		11.16	$E[VRP]$	20.67	20.97		21.20	21.49
$\sigma[VRP]$	16.23		15.13	$\sigma[VRP]$	16.03	17.59		20.73	21.87
$AC1(VRP)$	0.938		0.910	$AC1(VRP)$	0.419	0.569		0.559	0.664
$E[VIX^2]$		32.76		$E[VIX^2]$			36.78		
$\sigma[VIX^2]$		23.75		$\sigma[VIX^2]$			25.27		
$AC1(VIX^2)$		0.976		$AC1(VIX^2)$			0.756		
$E[RV]$	13.98		21.61	$E[RV]$			16.23		
$\sigma[RV]$	14.15		23.12	$\sigma[RV]$			17.04		
$AC1(RV)$	0.997		0.990	$AC1(RV)$			0.709		

Table 2: Short-Run Risk-Return Trade-Offs

The entries of the table are the slope coefficients as well as the coefficients of determination (R_l^2) of the regression

$$\frac{r_{t,t+l}}{l} = \alpha_{0l} + \beta_{1,0l} vp_t + \epsilon_{t,t+l}^{(0)}$$

where vp_t is the current variance premium and $r_{t,t+l}$ is the accumulated future returns over l months. In computing the daily variance premium, expected realized variance is a statistical forecast of realized variance using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The realized variance is the sum of squared 5-minute (RVFor) or the sum of squared daily (SQFor) log returns of the S&P 500 index over a 22-day period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$). In computing the monthly variance premium, expected realized variance is simply the lag realized variance or a statistical forecast of realized variance using an $AR(p)$ model. The realized variance is the sum of squared 5-minute log returns of the S&P 500 index over the period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$). All measures are on a monthly basis in percentage-squared.

l	1	3	6	9	12	l	1	3	6	9	12
	Daily data						Monthly data				
	Full Sample: January 1990 to December 2012										
	Forecast of RV based on RVFor						Forecast of RV is simply lag RV				
$\hat{\beta}_{1,0l}$	2.70	1.95	1.90	1.30	1.05	$\hat{\beta}_{1,0l}$	5.14	4.49	2.87	1.72	1.32
$se\left(\hat{\beta}_{1,0l}\right)$	1.26	1.24	0.68	0.56	0.53	$se\left(\hat{\beta}_{1,0l}\right)$	1.27	0.76	0.68	0.60	0.52
R_l^2	1.54	2.49	4.28	2.94	2.44	R_l^2	5.13	11.66	8.43	4.13	2.98
	Forecast of RV based on SQFor						Forecast of RV based on $AR(1)$				
$\hat{\beta}_{1,0l}$	4.36	3.31	2.13	1.18	0.83	$\hat{\beta}_{1,0l}$	3.53	3.48	2.87	1.96	1.54
$se\left(\hat{\beta}_{1,0l}\right)$	1.37	0.62	0.47	0.45	0.44	$se\left(\hat{\beta}_{1,0l}\right)$	1.68	1.03	0.66	0.63	0.61
R_l^2	3.39	6.07	4.49	1.99	1.24	R_l^2	1.89	6.33	7.98	5.29	4.03
	Subsample: January 1990 to October 2007										
	Forecast of RV based on RVFor						Forecast of RV is simply lag RV				
$\hat{\beta}_{1,0l}$	3.32	2.90	1.46	0.36	0.24	$\hat{\beta}_{1,0l}$	4.26	4.70	3.00	1.46	1.08
$se\left(\hat{\beta}_{1,0l}\right)$	1.56	1.15	1.07	1.10	1.08	$se\left(\hat{\beta}_{1,0l}\right)$	2.02	1.01	1.02	1.15	1.02
R_l^2	1.63	4.18	2.14	0.15	0.05	R_l^2	1.15	7.49	5.58	1.24	0.32
	Forecast of RV based on SQFor						Forecast of RV based on $AR(1)$				
$\hat{\beta}_{1,0l}$	3.95	3.15	1.59	0.53	0.35	$\hat{\beta}_{1,0l}$	3.76	3.89	2.20	0.69	0.23
$se\left(\hat{\beta}_{1,0l}\right)$	1.64	1.09	1.00	1.09	1.08	$se\left(\hat{\beta}_{1,0l}\right)$	1.80	1.14	1.09	1.23	1.08
R_l^2	2.00	4.27	2.21	0.32	0.13	R_l^2	0.99	5.90	3.34	-0.66	-1.33

Table 3: Long-Run Risk-Return Trade-Offs: January 1930 - December 2012

The entries of the table are the slope coefficients as well as the coefficients of determination (R^2) of the regression

$$\frac{r_{t,t+h}}{h} = \alpha_{mh} + \beta_{mh}\frac{\sigma^2_{t-m,t}}{m} + \epsilon^{(m)}_{t,t+h}$$

were $\sigma^2_{t-m,t}$ is the accumulated past monthly realized variance over the last m months and $r_{t,t+h}$ is the accumulated future monthly returns over the next h months. Standard errors are corrected for heteroskedasticity and autocorrelation based on the Newey and West (1987) procedure with $\max(m,h)$ lags.

	$m = h$									
h	1	2	3	4	5	6	7	8	9	10
$\hat{\beta}_{mh}$	0.38	0.55	0.41	-0.16	-0.31	-0.21	0.43	0.74	0.94	1.51
$se\left(\hat{\beta}_{mh}\right)$	0.44	0.32	0.36	0.31	0.43	0.54	0.52	0.51	0.60	0.62
R^2_{mh}	0.30	2.05	1.26	0.02	0.58	0.14	1.33	4.21	6.22	14.49

Table 4: Model Asset Pricing Moments

The entries of the table are the first and second moments of consumption and dividend growth rates, the first and second moments of the log price-dividend ratio, the log risk-free rate and excess log equity returns, and finally the slope and R^2 for the regression of 1-year, 3-year and 5-year future excess log equity returns onto the current log price dividend ratio. The first column represents annual data counterparts of these moments over the period from January 1930 to December 2012.

	Data	GDA	DA0	KP
δ		0.9989	0.9989	0.9989
γ		2.5	0	20
ψ		1.5	∞	1.5
ℓ		2.33	0.7	1
κ		0.989	1	1
s_σ/s_{2z}		0.125	0.125	0.125
$\phi_{1z}^{1/\Delta}$		0.995	0.995	0.995
k_{2z}		10	10	10
$\phi_{2z}^{1/\Delta}$		0.5	0.5	0.5
ρ		0.4043	0.4043	0.4043
$\beta_{\rho\sigma}$		0.07	0.07	0.07
$E[g_c]$	1.84	1.80	1.80	1.80
$\sigma[g_c]$	2.20	2.22	2.22	2.22
$AC1(g_c)$	0.48	0.25	0.25	0.25
$E[g_d]$	1.05	1.80	1.80	1.80
$\sigma[g_d]$	13.02	14.26	14.26	14.26
$AC1(g_d)$	0.11	0.25	0.25	0.25
$Corr(g_c, g_d)$	0.52	0.40	0.40	0.40
$E[pd]$	3.33	2.64	2.79	3.30
$\sigma[pd]$	0.45	0.27	0.42	0.06
$AC1(pd)$	0.85	0.96	0.96	0.96
$AC2(pd)$	0.75	0.90	0.90	0.90
$E[r_f]$	0.65	0.60	1.32	0.92
$\sigma[r_f]$	3.79	3.91	0.00	1.96
$E[r]$	5.35	8.62	7.21	4.59
$\sigma[r]$	20.17	20.55	22.94	17.75
$\beta(1Y)$	-0.11	-0.19	-0.12	-0.23
$R^2(1Y)$	3.53	6.49	4.78	0.62
$\beta(3Y)$	-0.09	-0.18	-0.11	-0.21
$R^2(3Y)$	16.01	16.43	12.75	1.63
$\beta(5Y)$	-0.09	-0.17	-0.11	-0.20
$R^2(5Y)$	23.75	23.32	18.89	2.40

Table 5: Model Variance Premium Moments

The entries of the table are the first and second moments of the options implied variance, the realized variance and the variance premium. The first column represents daily data counterparts of these moments over the period from January 1990 to December 2012. In computing the daily variance premium, expected realized variance is a statistical forecast of realized variance using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The realized variance is the sum of squared 5-minute (RVFor) or the sum of squared daily (SQFor) log returns of the S&P 500 index over a 22-day period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$). All measures are on a monthly basis in percentage-squared.

	SQFor (RVFor)	GDA	DA0	KP
δ		0.9989	0.9989	0.9989
γ		2.5	0	20
ψ		1.5	∞	1.5
ℓ		2.33	0.7	1
κ		0.989	1	1
s_σ/s_{2z}		0.125	0.125	0.125
$\phi_{1z}^{1/\Delta}$		0.995	0.995	0.995
k_{2z}		10	10	10
$\phi_{2z}^{1/\Delta}$		0.5	0.5	0.5
ρ		0.4043	0.4043	0.4043
$\beta_{\rho\sigma}$		0.07	0.07	0.07
$E\left[VIX^2\right]$ (1st)		44.93	46.52	30.38
$\sigma\left[VIX^2\right]$ (1st)		45.45	41.73	43.05
$E\left[VIX^2\right]$ (2nd)	39.84	56.95	47.42	30.95
$\sigma\left[VIX^2\right]$ (2nd)	40.23	69.02	66.34	67.37
$E[RV]$ (1st)		33.52	44.14	26.02
$\sigma[RV]$ (1st)		64.03	64.03	63.16
$E[RV]$ (2nd)	30.11 (19.37)	35.10	44.92	26.45
$\sigma[RV]$ (2nd)	52.53 (33.73)	67.13	65.19	64.02
$E[VRP]$ (1st)		11.41	2.38	4.36
$\sigma[VRP]$ (1st)		10.97	3.69	5.60
$E[VRP]$ (2nd)	9.73 (20.47)	21.85	2.50	4.50
$\sigma[VRP]$ (2nd)	20.19 (22.08)	2.13	2.99	3.60

Table 6: Model Short-Run Risk-Return Trade-Offs

The entries of the table are the slope coefficients as well as the coefficient of determination (R_l^2) of the regression

$$\frac{r_{t,t+l}}{l} = \alpha_{0l} + \beta_{1,0l} vp_t + \epsilon_{t,t+l}^{(0)}$$

where vp_t is the current monthly variance premium, and $r_{t,t+l}$ is the accumulated future monthly returns over l months. The first column represents daily data counterparts of these moments over the period from January 1990 to December 2012. In computing the daily variance premium, expected realized variance is a statistical forecast of realized variance using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The realized variance is The realized variance is the sum of squared 5-minute (RVFor) or the sum of squared daily (SQFor) log returns of the S&P 500 index over a 22-day period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$).

	SQFor (RVFor)	GDA	DA0	KP
δ		0.9989	0.9989	0.9989
γ		2.5	0	20
ψ		1.5	∞	1.5
ℓ		2.33	0.7	1
κ		0.989	1	1
s_σ/s_{2z}		0.125	0.125	0.125
$\phi_{1z}^{1/\Delta}$		0.995	0.995	0.995
k_{2z}		10	10	10
$\phi_{2z}^{1/\Delta}$		0.5	0.5	0.5
ρ		0.4043	0.4043	0.4043
$\beta_{\rho\sigma}$		0.07	0.07	0.07
$\beta_{1.01}$	4.36 (2.70)	47.57	0.24	15.55
R_1^2	3.39 (1.54)	2.94	0.00	1.19
$\beta_{1.02}$		36.65	-2.45	11.77
R_2^2		3.50	0.02	1.37
$\beta_{1.03}$	3.31 (1.95)	29.36	-4.24	9.25
R_3^2	6.07 (2.49)	3.37	0.11	1.27
$\beta_{1.04}$		24.35	-5.46	7.52
R_4^2		3.10	0.24	1.12
$\beta_{1.05}$		20.80	-6.31	6.29
R_5^2		2.82	0.40	0.98
$\beta_{1.06}$	2.13 (1.90)	18.20	-6.93	5.40
R_6^2	4.49 (4.28)	2.59	0.59	0.86
$\beta_{1.07}$		16.24	-7.39	4.72
R_7^2		2.40	0.78	0.77
$\beta_{1.08}$		14.73	-7.74	4.20
R_8^2		2.25	0.97	0.70
$\beta_{1.09}$	1.18 (1.30)	13.53	-8.01	3.79
R_9^2	1.99 (2.94)	2.14	1.17	0.64
$\beta_{1.010}$		12.57	-8.22	3.46
R_{10}^2		2.04	1.37	0.59
$\beta_{1.011}$		11.77	-8.39	3.18
R_{11}^2		1.97	1.58	0.55
$\beta_{1.012}$	0.83 (1.05)	11.10	-8.53	2.95
R_{12}^2	1.24 (2.44)	1.91	1.78	0.52

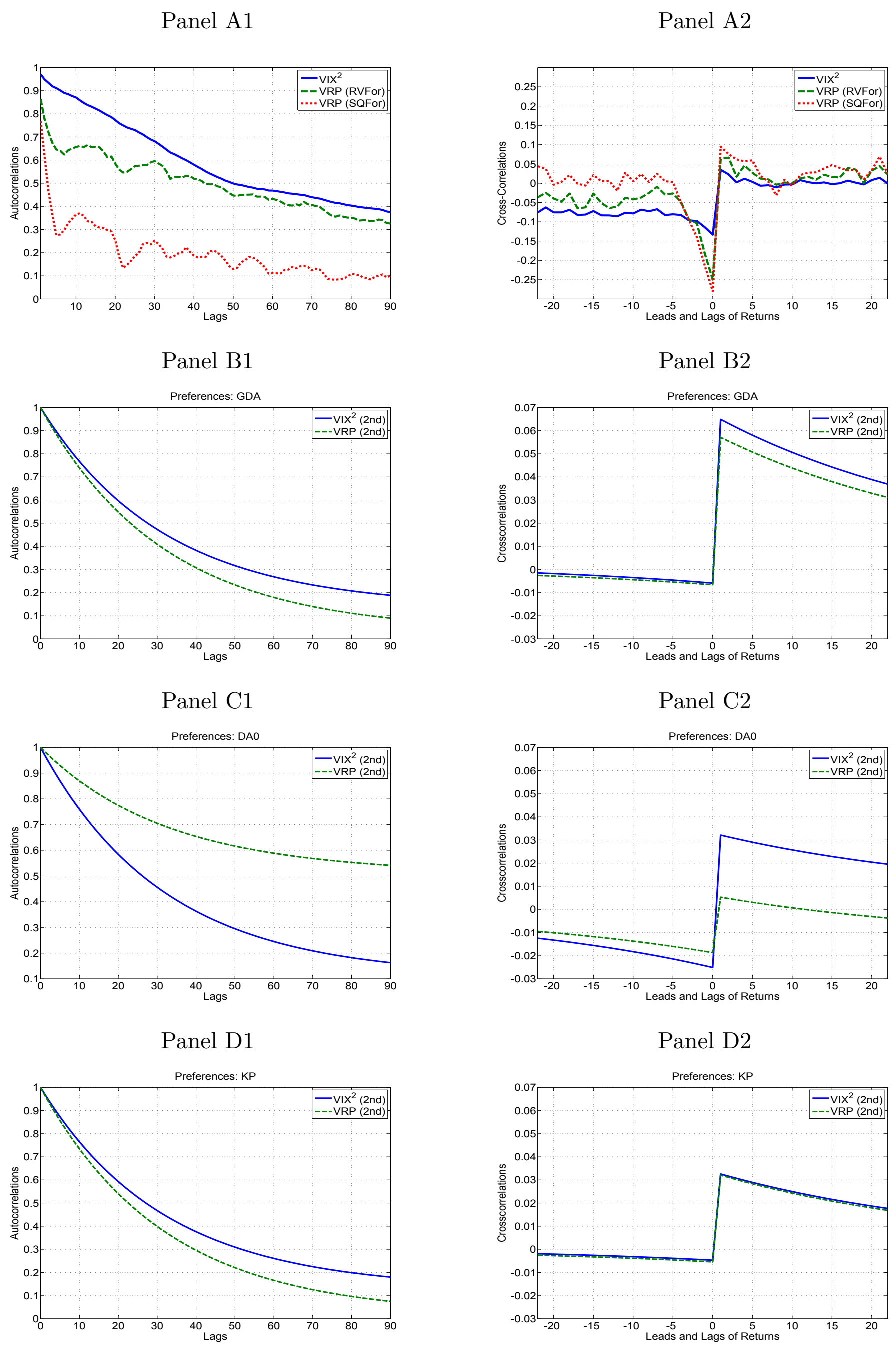
Table 7: Model Long-Run Risk-Return Trade-Offs

The entries of the table are the slope coefficients as well as the coefficients of determination (R^2) of the regression

$$\frac{r_{t,t+h}}{h} = \alpha_{hh} + \beta_{hh}\frac{\sigma^2_{t-h,t}}{h} + \epsilon^{(h)}_{t,t+h}$$

were $\sigma^2_{t-h,t}$ is the accumulated past monthly realized variance over the last h months and $r_{t,t+h}$ is the accumulated future monthly returns over the next h months. The first column represents data counterparts of these moments over the period from January 1930 to December 2012, where the monthly realized variance is computed as the sum of squared daily log returns of the S&P 500 index over the month.

	Data	GDA	DA0	KP
δ		0.9989	0.9989	0.9989
γ		2.5	0	20
ψ		1.5	∞	1.5
ℓ		2.33	0.7	1
κ		0.989	1	1
s_σ/s_{2z}		0.125	0.125	0.125
$\phi_{1z}^{1/\Delta}$		0.995	0.995	0.995
k_{2z}		10	10	10
$\phi_{2z}^{1/\Delta}$		0.5	0.5	0.5
ρ		0.4043	0.4043	0.4043
$\beta_{\rho\sigma}$		0.07	0.07	0.07
$\hat{\beta}_{11}$	0.38	0.31	0.08	0.18
R^2_{11}	0.30	1.36	0.25	0.30
$\hat{\beta}_{22}$	0.55	0.45	0.14	0.21
R^2_{22}	2.05	3.23	0.76	0.55
$\hat{\beta}_{33}$	0.41	0.55	0.19	0.24
R^2_{33}	1.26	5.35	1.42	0.83
$\hat{\beta}_{44}$	-0.16	0.62	0.23	0.25
R^2_{44}	0.02	7.35	2.15	1.09
$\hat{\beta}_{55}$	-0.31	0.67	0.26	0.26
R^2_{55}	0.58	9.10	2.88	1.31
$\hat{\beta}_{66}$	-0.21	0.70	0.29	0.27
R^2_{66}	0.14	10.56	3.58	1.50
$\hat{\beta}_{77}$	0.43	0.72	0.31	0.27
R^2_{77}	1.33	11.72	4.22	1.65
$\hat{\beta}_{88}$	0.74	0.73	0.32	0.27
R^2_{88}	4.21	12.61	4.79	1.77
$\hat{\beta}_{99}$	0.94	0.73	0.34	0.26
R^2_{99}	6.22	13.26	5.29	1.86
$\hat{\beta}_{10,10}$	1.51	0.73	0.34	0.26
$R^2_{10,10}$	14.49	13.71	5.71	1.92



The figure plots the data as well as the model-implied autocorrelations of the daily implied variance, realized variance and variance premium, as well as their cross-correlations with leads and lags of daily returns. In computing the daily variance premium in the data, expected realized variance is a statistical forecast of realized variance using the Heterogeneous Autoregressive model of Realized Variance (HAR-RV). The realized variance is the sum of squared 5-minute (RVFor) or the sum of squared daily (SQFor) log returns of the S&P 500 index over a 22-day period and its risk-neutral expectation is measured as the end-of-period VIX-squared de-annualized ($VIX^2/12$). The return series corresponds to excess returns on the S&P 500 index. The calibration of the consumption and dividends growths dynamics and of the preference parameter values corresponds to the benchmark case.

Figure 1: **Data and Model Volatility Effects**

Figure 2: Model Variance Premium Moments: GDA $\psi > 1$

The entries of the table are the first and second moments of the options implied variance, the realized variance and the variance premium. All measures are on a monthly basis in percentage-squared.

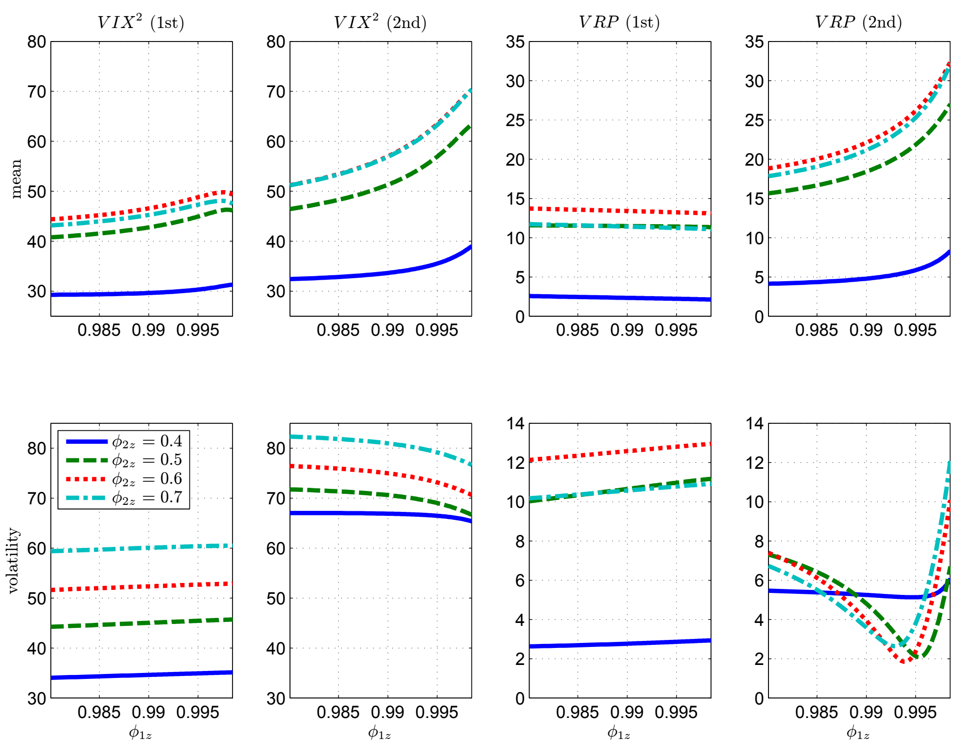


Figure 3: Model Short-Run Risk-Return Trade-Offs: GDA $\psi > 1$

The entries of the table are the slope coefficients as well as the coefficients of determination (R_l^2) of the regression

$$\frac{r_{t,t+l}}{l} = \alpha_{0l} + \beta_{1,0l} vp_t + \epsilon^{(0)}_{t,t+l}$$

where vp_t is the current monthly variance premium, and $r_{t,t+l}$ is the accumulated future monthly returns over l months.

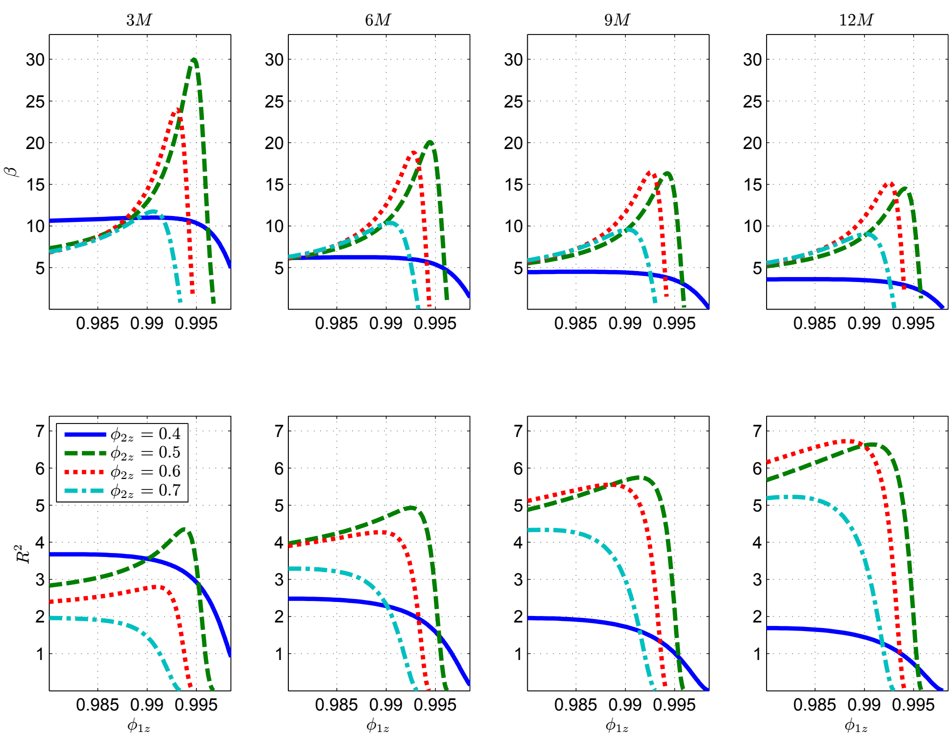


Figure 4: Model Long-Run Risk-Return Trade-Offs: GDA $\psi > 1$

The entries of the table are the slope coefficients as well as the coefficients of determination (R^2) of the regression

$$\frac{r_{t,t+h}}{h} = \alpha_{hh} + \beta_{hh}\frac{\sigma^2_{t-h,t}}{h} + \epsilon^{(h)}_{t,t+h}$$

were $\sigma^2_{t-h,t}$ is the accumulated past monthly realized variance over the last h months and $r_{t,t+h}$ is the accumulated future monthly returns over the next h months.